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Introducing Integers with Floats and Anchors

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Construct It!

Introducing Integers with Floats and Anchors

Use the Floats and Anchors context as well as physical and digital materials to help students understand integer addition and subtraction.

Christy Pettis and Aran Glancy

Over a decade ago, we were working with a team of sixth-grade teachers to revamp their integer unit. They had been using the "chip model" (i.e., red and yellow chips representing positive and negative numbers) to model integer addition and subtraction. While these teachers liked the hands-on aspect, their students were struggling to use the model to make sense of subtraction problems like ${}^{+}3 - {}^{-}5 = {}^{+}8$. In talking with the students, we learned that they disliked using the chip model for integer subtraction because they often needed to put in "zero pairs" of red and yellow chips. They found this confusing because they thought of zero as meaning "nothing."

To better help students reason about zero pairs and integer subtraction, we developed a series of activities based on adding and removing opposite objects to and from a boat. Similar to the credits and debts model proposed by Stephan and Akyuz (2012), Floats and Anchors uses the chip model, a vertical number line, and a real-world context to support reasoning about integer addition and subtraction (Figure 1). Adding a float to a boat makes it rise one foot (Figure 2a), while adding an anchor makes it sink one foot (Figure 2b). Similarly, removing the float or the anchor makes the boat sink or rise one foot, respectively.

In this article, we describe a sequence of four Floats and Anchors activities that we use to introduce integer addition and subtraction. This *Construct It!* task typically takes between 50 to 75 minutes of class time and requires no prior knowledge of integers. This task sets students up to successfully use both the chip model and a vertical number line as tools for reasoning about integers.

Figure 1 shows Floats and Anchors materials for classroom use. Boats are copied on transparencies to make it easier for students to see the number line as they move the ships up and down. Float and anchor pieces are copied on red and yellow cardstock and cut using a one-inch hole punch. All materials are available in the Printable Materials file (link online).

In Figure 2, floats make the boat move up one foot (a) and anchors make the boat move down one foot (b). GIFs showing these figures as animations can be found in the supplementary materials (link online).

ACTIVITY 1: MODELING ZERO IN THE INTEGERS (~10 MIN)

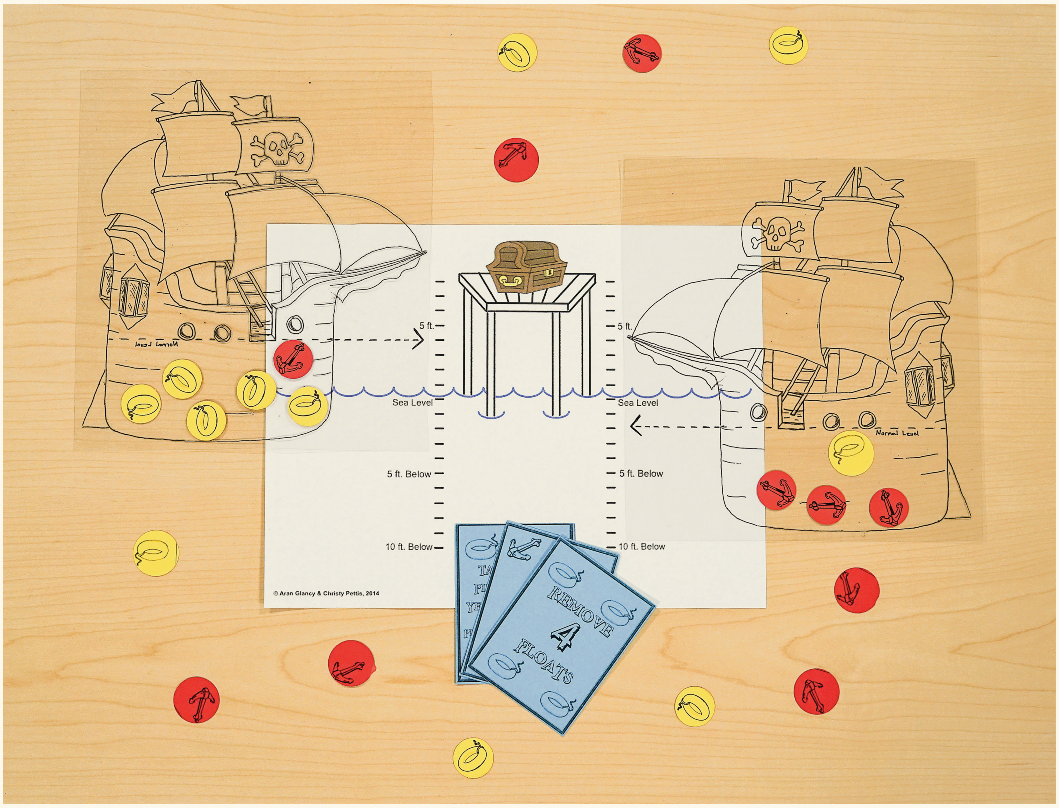
We begin the task by introducing the students to the mechanics of the boat and its "cargo" (i.e., the floats and anchors in the boat), and we ask students to practice putting in and taking out floats and anchors and to move the boat's position accordingly.

Next, we play What's Behind the Box. The goal of this activity is to allow students to explore how the introduction of opposites into our number system changes how we think about zero. We show a boat floating at sea level (zero) with some floats and anchors hidden behind a box and challenge students to correctly guess the hidden cargo (Figure 3). As students make guesses, we record their ideas as a list of “Ways of Representing 0” on the board (see Video 1). The more guesses they make, the clearer the key idea becomes: as long as there are equal numbers of floats and anchors, there are infinitely many ways to make zero.

After finishing the game, we highlight the idea that in the world of integers, zero represents “balance” rather than “nothing.” A boat with 10 floats and 10 anchors sits at zero, not because the floats and anchors go away (or “cancel out”), but because the floats and anchors create “balance.” In the integers, zero is not just “nothing” anymore!

In Figure 3, the two panels show the “What’s Behind the Box” game during play (a), and after students have guessed the correct number of floats and anchors in the boat (b). Slides to support this activity are available in the supplementary materials (link online).

Figure 1 Floats and Anchors Materials in Use in a Classroom



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ACTIVITY 2: MODELING OTHER INTEGERS (~10 MIN)

For the second activity, we ask students to move their boats to a number other than zero (e.g., $^{-}4$) and to consider different ways they could put floats and anchors in the boat to make the boat float at that position (Figure 4). The goal of this activity is to let students become familiar with the idea that opposites allow us to represent any integer in a multitude of ways. We use this activity to facilitate a whole-class discussion about equivalence in the integers, emphasizing that the many versions of $^{-}4$ (e.g., 0 floats and 4 anchors, 1 float and

Figure 2 Floats and Anchors Move the Boat Up and Down

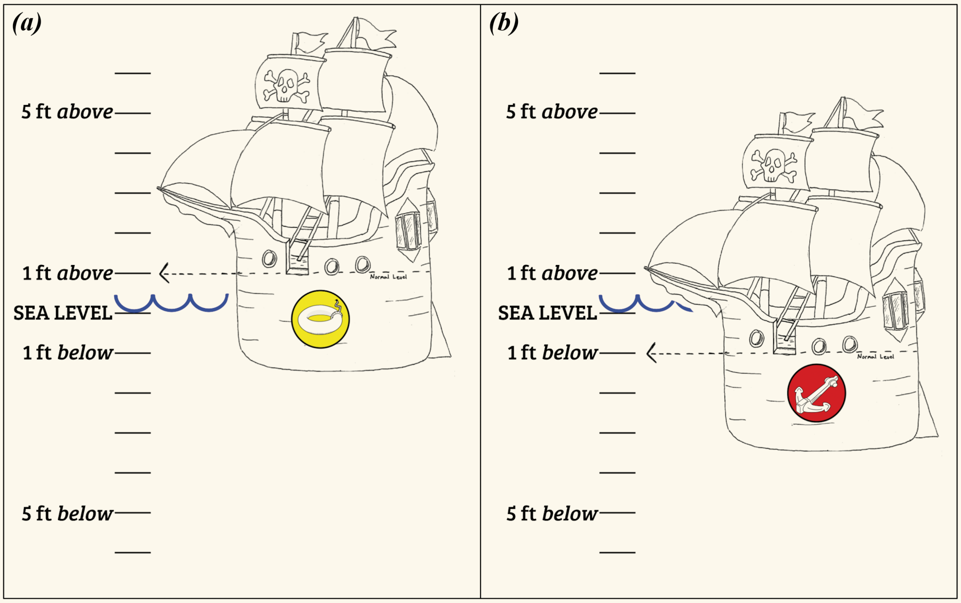
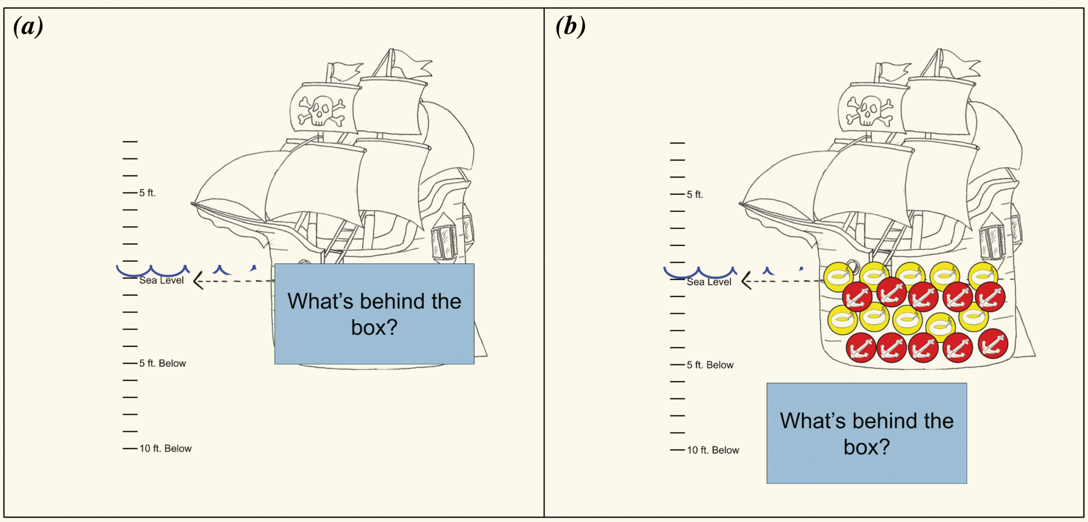


Figure 3 Sample Slide From the Interactive Slide Deck Before and After Gameplay



5 anchors, 10 floats and 14 anchors, and so on) all represent the same value, despite looking very different (see Video 1). Importantly, we do not tell students to remove the zero pairs from their boats to “prove” this equivalence, but instead emphasize that having opposites in our number system allows us to represent every integer in a multitude of ways.

Video 1 Representing Integers With Floats and Anchors

Watch the full video online

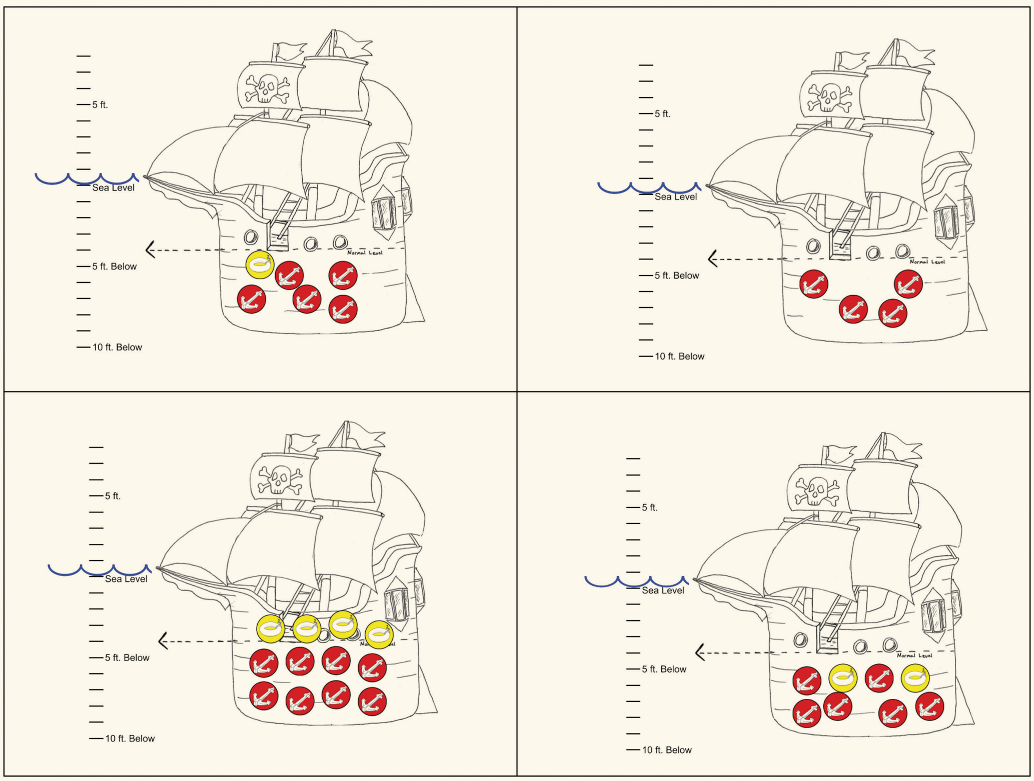
ACTIVITY 3: ADDING INTEGERS (~10 MIN)

Next, we move on to modeling addition. We ask the students to move their boat to a certain number, such as back to $^{-}4$, and encourage the students to find a way to make their boat float at $^{-}4$ in a way that is different from others around them. We then facilitate an investigation of what happens to the boat’s position when floats or anchors are added to the boat (see Figures 5 and 6). The goal of this activity is for students to discover that the addition of cargo can make the boat go up or down (i.e., adding can make the total increase or decrease), another new possibility created by the inclusion of negatives in our number system (see Video 2).

Figure 5 shows two versions of adding three floats to a boat starting at four below sea level. Although they have different numbers of floats and anchors on board, both boats start and end at the same position.

Figure 6 shows two versions of adding four anchors to a boat starting at one below sea level. Although they have different numbers of floats and anchors on board, both boats start and end at the same position.

Figure 4 Four Representations of $^{-}4$ Using Floats and Anchors



ACTIVITY 4: SUBTRACTING INTEGERS (~20 MIN)

Finally, we investigate what happens when we remove floats or anchors. We ask students to make their boats float at three above sea level ($^{+}3$) in a way that has both floats and anchors in the boat and is different from others around them. Similar to addition, one goal of this activity is for students to discover that the removal of cargo can make the boat go up or down, depending on what is removed from the boat. Another goal is to discuss the role that integer equivalence plays in modeling subtraction-as-removal problems with floats and anchors.

Video 2 Introducing Integer Addition with Floats and Anchors

Watch the full video online

Figure 5 Two Depictions of $^{-}4 + ^{+}3$ Using Floats and Anchors

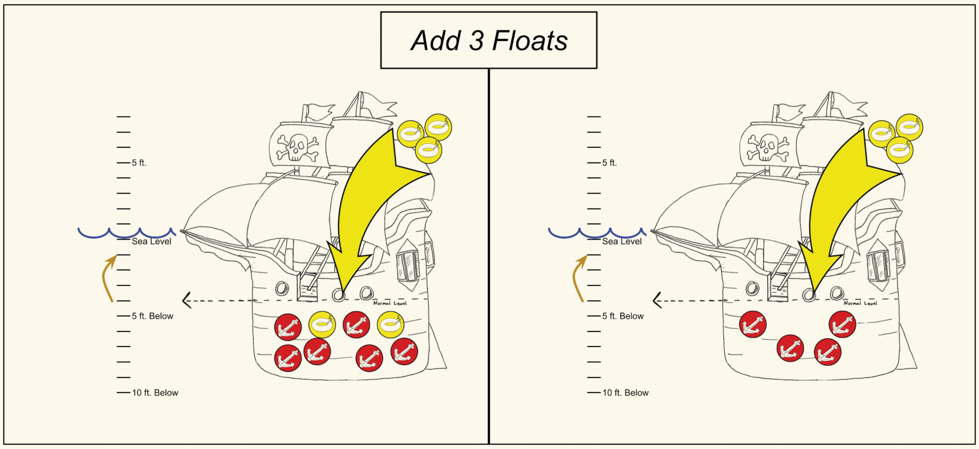
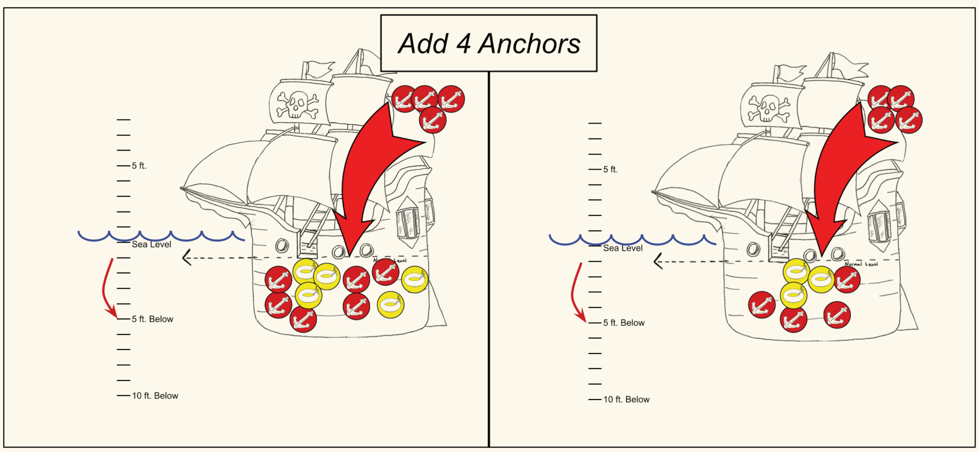


Figure 6 Two Depictions of $^{-}1 + ^{-}4$ Using Floats and Anchors



We start by asking all students to remove one float and to describe what happens to the ship's position. We revoice the key idea: "so if you start at positive three and take out one float, you end at positive two" while writing $^{+}3 - ^{+}1 = ^{+}2$ on the board. Next, we ask the students to remove one anchor. Again, we represent the situation with an equation, $^{+}2 - ^{-}1 = ^{+}3$, saying, "so if you start at positive two and take out one anchor, you end at positive three." Note, in this initial introduction, we do not explicitly discuss the different uses of the "-" symbol, but we are careful to write the negative symbol higher than the minus sign to help students visually distinguish them. We ask students to discuss: does *subtracting* a negative really lead to an *increase*?

Finally, we set up the key moment in the task by asking students to remove a large number of anchors, usually around five. We deliberately choose a number that will result in some students having enough anchors to remove while others will not have enough (Figure 7). Students without enough anchors invariably protest, "this is impossible!" Rather than resolving this conflict by asking students to "add float and anchor pairs," we instead ask those students who could model the problem to share their solution. We then ask the other students to "find a version of $^{+}3$ that allows them to solve the problem" and facilitate a discussion about how having equivalent representations allows them to solve any integer subtraction problem (see Video 3). Teachers can use the Task Sheet to close the task and check students' understanding.

Figure 7 demonstrates the key moment in the task: some students can follow the teacher's instruction to "remove five anchors" from a boat positioned at three above sea level, while others cannot.

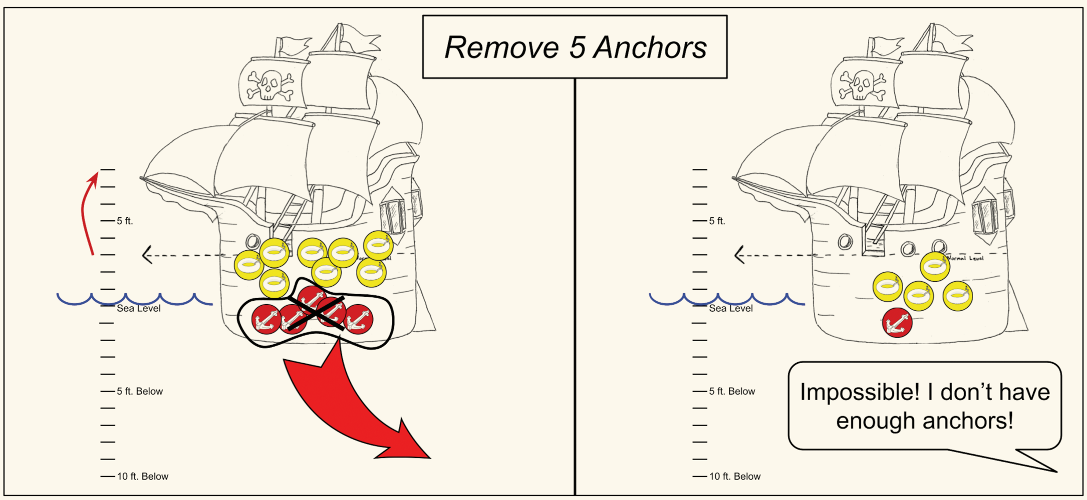
CONCLUSION

Research on integers indicates that subtracting negatives is challenging for many students (Bishop et al., 2022). We have found Floats and Anchors to be a particularly useful model for introducing integer

Video 3 Introducing Integer Subtraction With Floats and Anchors

Watch the full video online

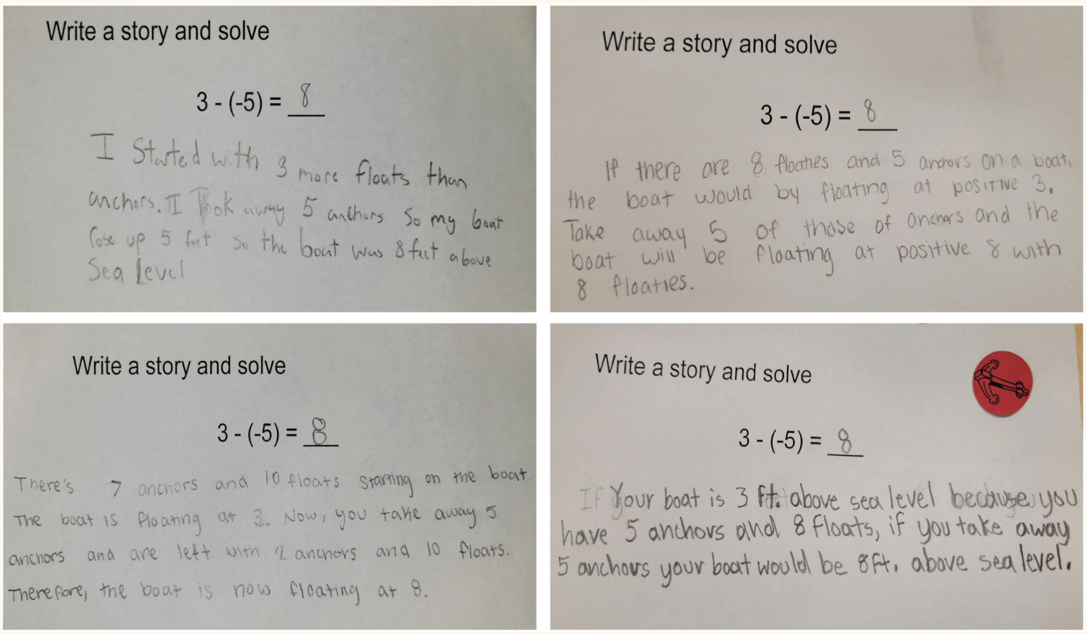
Figure 7 The Key Moment in the Lesson



subtraction as it includes tangible and intuitive representations of both positive and negative quantities. Crucially, the tangible representation of negative quantities as “anchors” allows students to model subtraction as removal and to intuitively grasp how removing anchors can have the same impact on the boat’s position as adding floats. In other words, the context gives them a way to make sense of why every subtraction problem involving integers can be thought of as an equivalent addition problem (e.g., $^{+}3 - {}^{-}5 = {}^{+}3 + {}^{+}5$). Similarly, the idea that the boat will stay floating at the same position as pairs of floats and anchors are added or removed gives them an “experientially real” (Stephan et al., 2020) way to make sense of the role that additive inverses play in allowing us to solve any subtraction problem in the integers. Once students gain this insight, they are ready to model any integer addition or subtraction problem (Figure 8).

No need for tricks or mnemonics: students can reason their way to the correct answer in ways consistent with the mathematics they are learning.

Figure 8 Typical Float and Anchor Stories Written by Middle School Students



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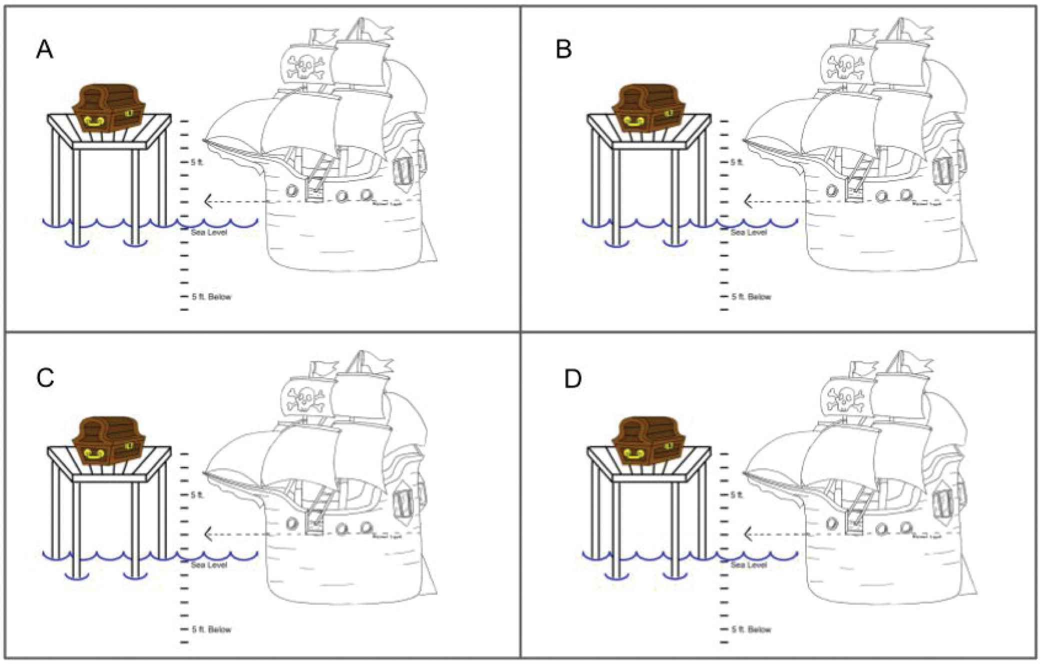
Subtracting Integers

1. How can you make the boats float at 2 feet above sea level? Draw different cargo (floats and anchors) in each of the four pictures to make the boat float at the correct position.



2. You are trying to use floats and anchors to solve $^{+}2 - {}^{-}3$. Which version(s) of 2 feet above sea level would be most helpful? Why?